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SOME SIMILARITY AND DISTANCE MEASURES ON FUZZY ROUGH SETS & IT'S APPLICATIONS

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Abstract

To establish similarity and dissimilarity between two groups or between two elements the similarity and distance measure has becomes an imperious tools. Many researchers proposed various similarity and distance measures by using different approaches. In which some are probabilistic in nature and other are non-probabilistic in nature. Here we discuss some non probabilistic similarity and distance measures by using fuzzy rough set approach. We also prove the validity of this proposed measure and discuss it application for decision making problem. We also compare proposed measures. The proposed measures can provide a useful tactic to measure the similarity and dissimilarity between fuzzy rough sets and between their elements.

Keywords:

fuzzy sets;
rough sets;
fuzzy rough sets;
similarity measures;
distance measures;
decision making.

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1 Introduction:

The real world is full of uncertainty, imprecision and vagueness in fields such as medical sciences, social sciences, engineering, economics etc. Classical set theory, which is based on the crisp and exact case may not be fully suitable for handling problems of uncertainty of such fields. So many authors have become interested in modeling uncertainty recently and have proposed various theories. Theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2], theory of vague sets [3] and theory of rough sets [4] are some of the well-known theories. In these theories, the concept such as cardinality, entropy, distance measure and similarity measure is widely used for the analysis and representation of various types of data information

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such as numerical information, interval-valued information, linguistic information and so on. The theory of fuzzy sets and rough sets are generalizations of vagueness and uncertainty. The concept of fuzzy set was introduced by Zadeh in his classical paper [1] in 1965. The concept of rough sets has been introduced and developed by Pawlak and co- workers [2, 5, 6] in deterministic and probabilistic sense. Although fuzzy sets and rough sets are methods to handle vague and inexact information, their starts and emphases are different. While some authors argued that for both theories, one theory is more general than other theory [3, 7], it is generally accepted that both theories are related but distinct and complementary theories [8, 9, 10]. The fuzzy sets emphasize on the morbid definition of the boundary of sets, in which the relations of "belonging to" and " not belong to" between elements and sets in the classical set theory are characterized by the membership degree; while rough sets use the approximation given the equivalent relation of the classical sets to research the indistinguishable of elements. Two theories of different points of view may complement each other. There are huge analyses on the relationship between rough sets and fuzzy sets [7, 8, 9 and 10]. For the combination of rough and fuzzy sets several suggestions have been made. The effect of these analyses directed towards the initiation of the notions of rough fuzzy sets and fuzzy rough sets [8, 11, 12, 13 and 14]. In [10] Yao state that a fuzzy rough set is originating from the approximation of a crisp set in a fuzzy approximation space. It is a duo of fuzzy sets in which the membership of an element is influenced by the degrees of similarity in all those elements in the set. Nanda and Majumdar [11] proposed fuzzy rough sets in 1992.

The measurement of uncertainty is an important topic for the theories dealing with uncertainty. The similarity measure, distance measure, entropy in fuzzy set theory and the relationship among these measures have been extensively studied for their wide applications in image processing, clustering, pattern recognition, case-based reasoning and many other fields. Kharal [15] introduced some set operation based on distance and similarity measures for soft sets. Moreover, the new similarity measures were applied to the problem of financial diagnosis of firms. On the basis of the distance measures between intuitionistic fuzzy set, Jiang et al. [16] proposed some distance measures between intuitionistic fuzzy soft sets and constructed some entropies on intuitionistic fuzzy soft sets and interval valued fuzzy soft sets. Wang and Qu [17] proposed a similarity measure, a distance measure and an entropy for vague soft set. Liu [18] investigated entropy, distance measure and similarity measure of fuzzy sets and their relations. Fan and Xie [19] introduced the similarity measure and fuzzy entropy induced by distance measure. Similarity measures based on union and intersection operations, the maximum difference and the difference and sum of membership grades are proposed by Pappis and Karacapilidis [20]. Wang [21] presented two similarity measures between fuzzy sets and between elements. Zhang et al [22, 23] proposed similarity measures for measuring the degree of similarity between vague sets and fuzzy rough sets. Later Niu Qi et al. [24] also intended a new similarity measures on fuzzy rough sets. Later Sharma and Gupta [25] proposed a sine trigonometric similarity measures on fuzzy rough sets and discussed it's application in medical diagnosis. Also Tiwari and Gupta [26] discussed cosine similarity measures for fuzzy sets, intuitionistic and interval-valued intuitionistic fuzzy-sets with application in medical diagnoses. Here in first section we give introduction, the rest of paper is summarized as:

In section 2, we discuss theoretical background of related concepts which is necessary for this paper. In third section we discussed some existing similarity measures which are used as a base of our proposed measures. In next section we proposed some distance and similarity measures between the elements of fuzzy rough sets and also show their validity in the form of theorem. In these measures some measures are trigonometric in nature. In section 5, we proposed similarity and distance measures between the fuzzy rough sets and show their validity. In next section we discussed an application of proposed measures related to decision making problem and also compare the proposed measures with the help of numerical example. At last we conclude the paper.

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2. Theoretical Ground:

In this section we discuss some related concepts and terms of this paper.

Fuzzy Sets: A fuzzy set F on U is characterized by a membership function $\mu_F(x) : U \to [0,1]$, as $F = \{x, \mu_F(x) : x \in U\}$, where U is Universe of discourse. Then $F^C = \{x, 1 - \mu_F(x) : x \in U\}$. where F^C is the complement of fuzzy set F.

The component-wise representation of fuzzy-set equality and inclusion are as:

$$F = G \iff \mu_F(x) = \mu_G(x), \quad \forall\, x \in U,$$

$$F \subseteq G \iff \mu_F(x) \leq \mu_G(x), \quad \forall\, x \in U,$$

In various descriptions of complement, intersection and union of fuzzy sets, we prefer the standard max-min system insinuated by Zadeh [1], in which fuzzy-set operations are outlined component-wise as:

$$\mu_{F^c} = 1 - \mu_F(x),$$

$$\mu_{F\cap G}(x) = min\{\mu_F(x), \mu_G(x)\},$$

$$\mu_{F\cup G}(x) = max\{\mu_F(x), \mu_G(x)\}.$$

A significant feature of fuzzy-set operations is that they are truth-functional. By using the membership functions of the fuzzy sets one can achieve membership functions of the complement, intersection and union of fuzzy sets.

Rough Sets: A brief recall of rough set is given in next definition:

Definition: Let U be a non-empty universe of discourse and R an equivalent relation on U, which is called an indistinguishable relation, $U/R = \{X_1, X_2, \ldots\ldots, X_n\}$ is all the equivalent class derived from R. $W = (U, R)$ are called an approximation space. $\forall X \subseteq U$, Suppose $\underline{X} = \{x \in U | [x] \subseteq X\}$ and $\overline{X} = \{x \in U | [x] \cap X \neq \emptyset\}$, a set pairs $(\underline{X}, \overline{X})$ are called a rough set in W, and symbolized as $X = (\underline{X}, \overline{X})$; $\underline{X}$ and $\overline{X}$ are the lower approximation and the upper approximation of X on W respecectively.

The strong and weak membership function of a rough set can be characterized by characteristic function of $\underline{X}$ and $\overline{X}$ respectively. Let the membership function of X and R indicated by μ_X and μ_R respectively. Then lower and upper approximations expressed by the following two expressions:

$$\mu_{\underline{R}(X)}(x) = inf\{\mu_X(y) \mid y \in U, (x, y) \in R\},$$

$$\mu_{\overline{R}(X)}(x) = sup\{\mu_X(y) \mid y \in U, (x, y) \in R\}, \qquad (2.1)$$

and

$$\mu_{\underline{R}(X)}(x) = inf\{1 - \mu_R(x, y) \mid y \notin A\},$$

$$\mu_{\overline{R}(X)}(x) = sup\{\mu_R(x, y) \mid y \in A\}, \qquad (2.2)$$

Here definition (2.2) is not described for sets $\emptyset$ and U. In these circumstances, we basically classify $\mu_{\underline{R}(U)}(x) = 1$ & $\mu_{\overline{R}(\emptyset)}(x) = 0, \forall x \in U$. In successive conversation, we will not clearly state these definitions of boundary cases. Based on the two equivalent definitions, lower and upper approximations may be interpreted as follows. An element $x \in \underline{X}$ if any element not in X is not equivalent to x, namely, $\mu_R(x, y) = 0$. An element $x \in \overline{X}$ if any element in X is equivalent to x, namely, $\mu_R(x, y) = 1$. These two aspects is significant in the combination of rough and fuzzy set. For convenience, the strong and weak membership function of a rough set can be precise as:

$$\mu_{\underline{R}(X)}(x) = inf\{\max[\mu_X(y), 1-\mu_R(x,y)] \mid y \in U\},$$
$$\mu_{\overline{R}(X)}(x) = sup\{\min[\mu_X(y), \mu_R(x,y)] \mid y \in U, (x,y) \in R\}, \qquad (2.3)$$

For reference sets $X \cap Y$ and $X \cup Y$, $\left(\underline{R}(X \cap Y), \overline{R}(X \cap Y)\right)$ and $\left(\underline{R}(X \cup Y), \overline{R}(X \cup Y)\right)$ are the intersection and union of two rough sets $\left(\underline{R}(X), \overline{R}(X)\right)$ and $\left(\underline{R}(Y), \overline{R}(Y)\right)$, respectively. With a reference set X^c, the complement of rough-set is defined by $\left(\underline{R}(X^c), \overline{R}(X^c)\right)$.

Fuzzy Rough Sets: The approximation of a crisp set in a fuzzy approximation space is called a fuzzy rough set. We exclaim the pair of fuzzy set $\left(\underline{R}(X), \overline{R}(X)\right)$ a fuzzy rough set with reference set $X \subseteq U$. A fuzzy rough set is characterized by a crisp set and two fuzzy sets:

$$\mu_{\underline{R}(X)}(x) = inf\{1-\mu_R(x,y) \mid y \notin A\},$$
$$\mu_{\overline{R}(X)}(x) = sup\{\mu_R(x,y) \mid y \in A\},$$

Definition: Let S be the set of the whole rough sets, $A = (\underline{A}, \overline{A}) \in S$, then a fuzzy rough set $X = (\underline{X}, \overline{X})$ in A can be expressed by a pair mapping $\mu_{\underline{X}}$, $\mu_{\overline{X}}$

$$\mu_{\underline{X}} \; : \; \underline{X} \to [0,1], \qquad \mu_{\overline{X}} \; : \; \overline{X} \to [0,1].$$

Also $\mu_{\underline{X}} \leq \mu_{\overline{X}}$, $\forall x \in \dot{\overline{X}}$. And then, a fuzzy rough set X in A could be signified by

$$X = \left\{\langle x, \mu_{\underline{X}}, \mu_{\overline{X}}\rangle \mid \forall x \in \overline{X}\right\}. \qquad (2.4)$$

and $\left\{\langle \mu_{\underline{X}}, \mu_{\overline{X}}\rangle \mid \forall x \in X\right\}$ is called the value of fuzzy rough of x in A, still written as x.

Suppose X is a fuzzy rough set in A, when A is a finite set, then

$$X = \sum_{i=1}^{n} \langle x, \mu_{\underline{X}}(x_i), \mu_{\overline{X}}(x_i)\rangle \mid x_i, x_i \in \overline{X}.$$

When A is continuous, then

$$X = \int \langle \mu_{\underline{X}}, \mu_{\overline{X}}\rangle \mid x, \forall x \in \overline{X}. \; over \; A.$$

The whole fuzzy rough set in A is renowned by $F^R(X)$.

Let $X = (\underline{X}, \overline{X})$ be a fuzzy rough set in A, $X^c = \left(\underline{X}^c, \overline{X}^c\right)$ is called the complementary set of $X = (\underline{X}, \overline{X})$, where $\mu_{\underline{X}^c}(x) = 1 - \mu_{\underline{X}}(x)$, $\forall x \in \underline{X}$; $\mu_{\overline{X}^c}(x) = 1 - \mu_{\overline{X}}(x)$, $\forall x \in \overline{X}$.

The order relation in X is defined by the following condition:

$$x \leq y \Leftrightarrow \mu_{\underline{X}}(x) \leq \mu_{\underline{X}}(y) \; and \; \mu_{\overline{X}}(x) \leq \mu_{\overline{X}}(y).$$

Similarity Measures: A similarity measure or similarity function is a real-valued function that enumerates the similarity between two objects. Although, no specific definition of a similarity measures subsisted, usually such measures are some implication of the inverse of distance measures. Similarity measures are exploited in system configuration. Higher scores are given to more-similar quality, and lower or negative scores for dissimilar quality.

A similarity measure is an authoritative measure if it convinced following condition:

1. $0 \leq S(A,B) \leq 1$

2. $S(A, B) = 1$ (or maximum similarity) if and only if $A = B$.
3. $S(A, B) = 0$ if and only if $B = 1 - A$ or (A^c).
4. $S(A, B) = S(B, A)$ for all A and B, where $S(A, B)$ is the similarity between data objects A and B.
5. If $A \subseteq B \subseteq C$, then $S(A, B) \geq S(A, C)$, and $S(B, C) \geq S(A, C)$.

Distance Measures: A distance measure or distance function is a real-valued function that enumerates the dissimilarity between two objects. Although, no specific definition of a distance measures subsisted, usually such measures are some implication of the inverse of similarity measures. Distance measures are exploited in system configuration. Higher scores are given to more-dissimilar quality, and lower or negative scores for similar quality.

A distance measure is an authoritative measure if it convinced following condition:

1. $0 \leq D(A, B) \leq 1$
2. $D(A, B) = 0$ (or maximum dissimilar) if and only if $A = B$.
3. $D(A, B) = 1$ if and only if $B = 1 - A$ or (A^c).
4. $D(A, B) = D(B, A)$ for all A and B, where $D(A, B)$ is the measure of dissimilarity between data objects A and B.
5. If $A \subseteq B \subseteq C$, then $D(A, C) \geq D(A, B)$, and $D(A, C) \geq S(B, C)$.

It is easy to see that (5) is equivalent to 6: If $A \subseteq B \subseteq C \subseteq D$, then $D(A, D) \geq D(B, C)$.

Proposition 1: There exists a one-to-one correlation between all distance measures and all similarity measures and a distance measure D and its corresponding similarity measure S satisfy $D + S = 1$.

Because distance and similarity measures are complementary concepts, similarity measures can be used to define distance measures and vice-versa. Thus $S = 1 - D$ is called the similarity measure generated by distance measure D and is denoted by $S\langle D \rangle$ and $D = 1 - S$ the distance measure generated by similarity measure S and is denoted by $D\langle S \rangle$.

3. Some Similarity Measures:

In recent years, various researchers and authors recommended and researched different similarity measure of fuzzy sets and Intuitionistic fuzzy sets [15-27]. Chen [27] is one that gave the similarity measure related to intuitionistic fuzzy set for measuring the degree of similarity between elements as follows.

Definition. Let $x = [t_A(x), 1 - f_A(x)]$ and $y = [t_A(y), 1 - f_A(y)]$ be two fuzzy values in IFS A. A degree of similarity between the fuzzy values x and y can be estimated by the function S_C,

$$S_C(x, y) = 1 - \frac{1}{2}|M(x) - M(y)|, \qquad (3.1)$$

Where $M(x) = t_A(x) - f_A(x)$ and $M(y) = t_A(y) - f_A(y)$.

Definition. Let $x = [\, t_A(x), 1 - f_A(x)]$ and $y = [t_A(y), 1 - f_A(y)]$ be two fuzzy values in IFS A. A degree of similarity between the fuzzy values x and y can be calculated by the function S_H,

$$S_H(x, y) = 1 - \frac{1}{2}(|t_A(x)| - |t_A(y)| + |f_A(x)| - |f_A(y)|) \qquad (3.2)$$

In [22], Zhang provide a similarity measure between two fuzzy rough sets and fuzzy rough values as follows.

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Definition. Let A be a fuzzy rough set in X, $x = \langle \underline{\mu_A(x)},\ \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)},\ \overline{\mu_A(y)} \rangle$ the fuzzy rough values in A. The degree of similarity between the fuzzy rough values x and y can be assessed by the function S_Z

$$S_Z(x, y) = 1 - \frac{1}{2}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right) \qquad (3.3)$$

On the basis of fuzzy information handling he identified some axioms or rule which assured the authenticity of similarity measures of fuzzy rough values. Consider $x = \langle \underline{\mu_A(x)},\ \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)},\ \overline{\mu_A(y)} \rangle$ *and* $z = \langle \underline{\mu_A(z)},\ \overline{\mu_A(z)} \rangle$ be the fuzzy rough values in a fuzzy rough set A. Then S is the similarity measure to quantify the degree of similarity between elements in A, if persuade following conditions:

1. (symmetry) $S(x, y) = S(y, x)$.
2. (monotony) if $x \le y \le z$, then $S(x, z) \le min\{S(x, y), S(y, z)\}$.
3. $S(x, y) = 0$ iff $(x = \langle 0, 0 \rangle)$ and $(y = \langle 1, 1 \rangle)$; or $(x = \langle 1, 1 \rangle)$ and $(y = \langle 0, 0 \rangle)$;
   $S(x, y) = 1$ iff $\left(\underline{\mu_A(x)} = \underline{\mu_A(y)}\right)$ and $\left(\overline{\mu_A(x)} = \overline{\mu_A(y)}\right)$.
4. $S(x, y) = S(x^c, y^c)$.
5. $\forall x \in X$, if $S(x, y) = S(x, z) \Longrightarrow S(y, z) = 1$.

Corresponding to the condition of similarity measure between the elements of fuzzy rough sets we define the condition or necessary axioms for valid distance measure between the elements of fuzzy rough set A as:

1. $D(x, y) = D(y, x)$, symmetric in nature,
2. If $x \le y \le z$, then $D(x, z) \ge max\{D(x, y), D(y, z)\}$,
3. $D(x, y) = 1$ iff $(x = \langle 0, 0 \rangle)$ and $(y = \langle 1, 1 \rangle)$; or $(x = \langle 1, 1 \rangle)$ and $(y = \langle 0, 0 \rangle)$;
   $D(x, y) = 0$ iff $\left(\underline{\mu_A(x)} = \underline{\mu_A(y)}\right)$ and $\left(\overline{\mu_A(x)} = \overline{\mu_A(y)}\right)$.
4. $D(x, y) = D(x^c, y^c)$.
5. $\forall x \in X$, if $D(x, y) = D(x, z) \Longrightarrow D(y, z) = 0$.

Here the order relations of the fuzzy rough value as follow:

$$x \le y \Leftrightarrow \left(\underline{\mu_A(x)} \le \underline{\mu_A(y)}\right) \ and \ \left(\overline{\mu_A(x)} \le \overline{\mu_A(y)}\right)$$

In 2008 Qi et al. [24] proposed a similarity measures between fuzzy rough sets and its elements. The similarity measures defined by them as:

Definition: Let $A \in F^R(X)$ and τ_x, ρ_x, σ_x as above. $x = \langle \underline{\mu_A(x)},\ \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)},\ \overline{\mu_A(y)} \rangle$ in A. The similarity degree between x and y can be evaluated by the function S,

$$S(x, y) = 1 - \frac{1}{2}(\rho_{xy} + \sigma_{xy}), \qquad (3.4)$$

where $\rho_{xy} = |\rho_x - \rho_y|$ and $\sigma_{xy} = |\sigma_x - \sigma_y|$.

Definition: let $A \in F^R(X)$ and $x = \langle \underline{\mu_A(x)},\ \overline{\mu_A(x)} \rangle \in A$. Then:

1. $\tau_x = \overline{\mu_A(x)} - \underline{\mu_A(x)}$ is called the degree of indeterminacy of the element $x \in A$.
2. $\rho_x = \underline{\mu_A(x)} + \tau_x \underline{\mu_A(x)} = (1 + \tau_x)\underline{\mu_A(x)}$ is called the degree of favor $x \in A$
3. $\sigma_x = 1 - \overline{\mu_A(x)} + \tau_x \left(1 - \overline{\mu_A(x)}\right)$ is called the degree of against $x \in A$.

Remarks. (1) x is more unspecified for superior value of τ_x. If $\tau_x = 1, i.e. \overline{\mu_A(x)} = 1$ *and* $\underline{\mu_A(x)} = 0$, then we know nothing for x; if $\forall x \in A, \tau_x = 0$, then the fuzzy rough set A is a fuzzy set; if $\forall x \in A$, $\overline{\mu_A(x)} = \underline{\mu_A(x)} = 1(0)$, then the fuzzy rough set A is a common set.

Corresponding to similarity measure – (3.4) of the elements of Fuzzy rough set Sharma and Gupta [25] find sine trigonometric similarity measures of fuzzy rough sets elements as:

$$S_{sin}(x,y) = \sin\left[\frac{\pi}{2}\left(1 - \frac{1}{2}(\rho_{xy} + \sigma_{xy})\right)\right], \qquad (3.5)$$

4. Some New Similarity & Distance Measures between the Elements of Fuzzy Rough Sets:

In this section we proposed some new similarity and distance measures between the elements of fuzzy rough sets by using the existing measures and also by applying proposition 1 stated above.

Corresponding to similarity measures – (3.3) we have distance measure by using proposition 1

$$D_{O1}(x,y) = \frac{1}{2}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right) \qquad (4.1)$$

Corresponding to similarity measures –(3.3) we have sine and tan trigonometric similarity measures

$$S_{osin\,1}(x,y) = \sin\left[\frac{\pi}{2}\langle 1 - \frac{1}{2}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right)\ \rangle\right] \qquad (4.2(a))$$

The similarity measure defined in 4.2 (a) can be represented in the form of cosine trigonometric similarity measure as:

$$S_{ocos\,1}(x,y) = \cos\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right)\right] \qquad (4.2(b))$$

$$S_{otan\,1}(x,y) = \tan\left[\frac{\pi}{4}\langle 1 - \frac{1}{2}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right)\ \rangle\right] \qquad (4.3)$$

Corresponding to distance measure (4.1) we have sine and tan trigonometric distance measures

$$D_{O1sin}(x,y) = \sin\left[\frac{\pi}{2}\langle\frac{1}{2}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right)\rangle\right]$$

$$D_{O1sin}(x,y) = \sin\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right)\right] \qquad (4.4)$$

$$D_{O1tan}(x,y) = \tan\left[\frac{\pi}{8}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right)\right] \qquad (4.5)$$

Corresponding to similarity measures – (3.4) we have distance measure by using proposition 1

$$D_{o2}(x,y) = \frac{1}{2}(\rho_{xy} + \sigma_{xy}), \qquad (4.6)$$

Corresponding to similarity measures –(3.4) we have tan and cosine trigonometric similarity measures

$$S_{otan\,2}(x,y) = \tan\left[\frac{\pi}{4}\left(1 - \frac{1}{2}(\rho_{xy} + \sigma_{xy})\right)\right], \qquad (4.7)$$

$$S_{ocos\,2}(x,y) = \cos\left[\frac{\pi}{4}(\rho_{xy} + \sigma_{xy})\right], \qquad (4.8)$$

Here measure –(3.5) is equivalent to –(4.8) measure

Corresponding to distance measure (4.6) we have sine and tan trigonometric distance measures

$$D_{osin\,2}(x,y)=\sin\left[\frac{\pi}{4}\left(\rho_{xy}+\sigma_{xy}\right)\right], \qquad (4.9)$$

$$D_{otan\,2}(x,y)=\tan\left[\frac{\pi}{8}\left(\rho_{xy}+\sigma_{xy}\right)\right], \qquad (4.10)$$

where $\rho_{xy}=\left|\rho_x-\rho_y\right|$ and $\sigma_{xy}=\left|\sigma_x-\sigma_y\right|$.

Now we prove the validity of these proposed measures in the form of theorem by satisfying their axioms defined above.

Theorem 1: Let A be a fuzzy rough set in X, $x=\langle\underline{\mu_A(x)},\ \overline{\mu_A(x)}\rangle$, $y=\langle\underline{\mu_A(y)},\ \overline{\mu_A(y)}\rangle$ the fuzzy rough values in A. Then the measure of the degree of dissimilarity between the fuzzy rough values x and y defined below is a valid measures.

$$D_{O1}(x,y)=\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)$$

Proof: Since above measure is defined from the valid similarity measure defined below by using proposition 1.

$$S_Z(x,y)=1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)$$

Thus it is trivial that proposed distance measure is valid.

Theorem 2: Let A be a fuzzy rough set in X, $x=\langle\underline{\mu_A(x)},\ \overline{\mu_A(x)}\rangle$, $y=\langle\underline{\mu_A(y)},\ \overline{\mu_A(y)}\rangle$ the fuzzy rough values in A. Then the measure of the degree of dissimilarity between the fuzzy rough values x and y defined below is a valid measures.

$$D_{o2}(x,y)=\frac{1}{2}\left(\rho_{xy}+\sigma_{xy}\right)$$

Proof: Since above measure is defined from the valid similarity measure defined below by using proposition 1.

$$S(x,y)=1-\frac{1}{2}\left(\rho_{xy}+\sigma_{xy}\right)$$

Thus it is trivial that proposed distance measure is valid.

Theorem 3: Let A be a fuzzy rough set in X, $x=\langle\underline{\mu_A(x)},\ \overline{\mu_A(x)}\rangle$, $y=\langle\underline{\mu_A(y)},\ \overline{\mu_A(y)}\rangle$ the fuzzy rough values in A. Then the following measures of the degree of dissimilarity between the fuzzy rough values x and y defined below are valid measures.

a. $D_{O1sin}(x,y)=\sin\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\right]$
b. $D_{O1tan}(x,y)=\tan\left[\frac{\pi}{8}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\right]$

Proof: First we prove the validity of part (a) to prove the validity of this part we first prove following lemmas:

Lemma 1: Let A be a fuzzy rough set in X, $x=\langle\underline{\mu_A(x)},\ \overline{\mu_A(x)}\rangle$, $y=\langle\underline{\mu_A(y)},\ \overline{\mu_A(y)}\rangle$ $and\ z=\langle\underline{\mu_A(z)},\ \overline{\mu_A(z)}\rangle$ the fuzzy rough values in A and $D_{O1sin}(x,y)$ as above. If $x\le y\le z$ then $D_{O1sin}(x,z)\ge\max\{D_{O1sin}(x,y),D_{O1sin}(y,z)\}$.

Proof: Since in theorem 1 we prove that the measure $D_{01}(x,y)$ is a valid measure so we have $\left(\left|\underline{\mu_A(x)}-\right.\right.$
μA(z)+μA(x)−μA(z)≥maxμAx−μAy+μAx−μAy, μA(y)−μA(z)+μA(y)−μA(z)

Since sine is an increasing function in interval $[0, \pi/2]$, so we have $\sin\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(z)}\right|+\right.\right.$
μA(x)−μA(z)≥maxsinπ4μA(x)−μA(y)+μA(x)−μA(y), sinπ4μA(y)−μA(z)+μA(y)−μA(z)

Thus we have $D_{01sin}(x,z) \geq \max\{D_{01sin}(x,y), D_{01sin}(y,z)\}$.

Lemma 2: Let A be a fuzzy rough set in X, $x = \langle\underline{\mu_A(x)}, \overline{\mu_A(x)}\rangle$, $y = \langle\underline{\mu_A(y)}, \overline{\mu_A(y)}\rangle$ the fuzzy rough values in A and $D_{01sin}(x,y)$ as above, then

a) $D_{01sin}(x,y) = 1$ iff $(x = \langle 0,0\rangle)$ *and* $(y = \langle 1,1\rangle)$; or $(x = \langle 1,1\rangle)$ *and* $(y = \langle 0,0\rangle)$;

b) $D_{01sin}(x,y) = 0$ iff $\left(\underline{\mu_A(x)} = \underline{\mu_A(y)}\right)$ *and* $\left(\overline{\mu_A(x)} = \overline{\mu_A(y)}\right)$.

Proof: Since in theorem 1, we prove that the measure $D_{01}(x,y)$ is a valid measure so for first part for this lemma we have, $\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right) = 2$. Thus we have, $\sin\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\right.\right.$
*μA(x)−μA(y)=*sin*π2=1.* Hence *DO1sinx, y=1* iff *x=0, 0 and y=1, 1;* or *x=1, 1 and y=0, 0.*

Now for second part of this lemma we have $\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right) = 0$. Thus we have $\sin\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\right] = 0$. Hence $D_{01sin}(x,y) = 0$ iff $\left(\underline{\mu_A(x)} = \underline{\mu_A(y)}\right)$ *and* $\left(\overline{\mu_A(x)} = \overline{\mu_A(y)}\right)$.

Lemma 3: Let A be a fuzzy rough set in X, $x = \langle\underline{\mu_A(x)}, \overline{\mu_A(x)}\rangle$, $y = \langle\underline{\mu_A(y)}, \overline{\mu_A(y)}\rangle$ the fuzzy rough values in A and $D_{01sin}(x,y)$ as above, then

a. $D_{01sin}(x,y) = D_{01sin}(y,x)$,
b. $D_{01sin}(x,y) = D_{01sin}(x^c, y^c)$

Proof: Since in theorem1, we prove that the measure $D_{01}(x,y)$ is a valid measure so lemma is trivially proved.

Lemma 4: Let A be a fuzzy rough set in X, $x = \langle\underline{\mu_A(x)}, \overline{\mu_A(x)}\rangle$, $y = \langle\underline{\mu_A(y)}, \overline{\mu_A(y)}\rangle$ *and* $z = \langle\underline{\mu_A(z)}, \overline{\mu_A(z)}\rangle$ the fuzzy rough values in A and $D_{01sin}(x,y)$ as above. If $\forall x \in \overline{A}, D_{01sin}(x,y) = D_{01sin}(x,z)$ then $D_{01sin}(y,z) = 0$.

Proof: Since in theorem 1, we prove that the measure $D_{01}(x,y)$ is a valid measure so we have $\left(\left|\underline{\mu_A(y)}-\right.\right.$
μA(z)+μA(y)−μA(z)=0. Thus we have sin*π4μA(y)−μA(z)+μA(y)−μA(z)=0.* Hence *DO1siny, z=0,* $\forall x \in \overline{A}, D_{01sin}(x,y) = D_{01sin}(x,z)$.

Hence proposed measure is valid measures

Similarly we prove the validity of part (b)

Theorem 4: Let A be a fuzzy rough set in X, $x = \langle \underline{\mu_A(x)}, \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)}, \overline{\mu_A(y)} \rangle$ the fuzzy rough values in A. Then the following measures of the degree of dissimilarity between the fuzzy rough values x and y defined below are valid measures.

a. $D_{osin\,2}(x, y) = \sin\left[\frac{\pi}{4}\left(\rho_{xy} + \sigma_{xy}\right)\right]$,
b. $D_{otan\,2}(x, y) = \tan\left[\frac{\pi}{8}\left(\rho_{xy} + \sigma_{xy}\right)\right]$,

Proof: First we prove the validity of part (a) to prove the validity of this part we first prove following lemmas:

Lemma 1: Let A be a fuzzy rough set in X, $x = \langle \underline{\mu_A(x)}, \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)}, \overline{\mu_A(y)} \rangle$ and $z = \langle \underline{\mu_A(z)}, \overline{\mu_A(z)} \rangle$ the fuzzy rough values in A and $D_{O2sin}(x, y)$ as above. If $x \le y \le z$ then $D_{O2sin}(x, z) \ge \max\{D_{O2sin}(x, y), D_{O2sin}(y, z)\}$.

Proof: Since in theorem 2 we prove that the measure $D_{O2}(x, y)$ is a valid measure so we have $(\rho_{xz} + \sigma_{xz}) \ge \max\left\{\begin{matrix}(\rho_{xy} + \sigma_{xy}),\\ (\rho_{yz} + \sigma_{yz})\end{matrix}\right\}$

Since sine is an increasing function in interval $[0, \pi/2]$, so we have $\sin\left[\frac{\pi}{4}(\rho_{xz} + \sigma_{xz})\right] \ge \max\left\{\begin{matrix}\sin\left[\frac{\pi}{4}(\rho_{xy} + \sigma_{xy})\right],\\ \sin\left[\frac{\pi}{4}(\rho_{yz} + \sigma_{yz})\right]\end{matrix}\right\}$

Thus we have $D_{O2sin}(x, z) \ge \max\{D_{O2sin}(x, y), D_{O2sin}(y, z)\}$.

Lemma 2: Let A be a fuzzy rough set in X, $x = \langle \underline{\mu_A(x)}, \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)}, \overline{\mu_A(y)} \rangle$ the fuzzy rough values in A and $D_{O2sin}(x, y)$ as above, then

a) $D_{O2sin}(x, y) = 1$ iff $(x = \langle 0, 0 \rangle)$ and $(y = \langle 1, 1 \rangle)$; or $(x = \langle 1, 1 \rangle)$ and $(y = \langle 0, 0 \rangle)$;
b) $D_{O2sin}(x, y) = 0$ iff $\left(\underline{\mu_A(x)} = \underline{\mu_A(y)}\right)$ and $\left(\overline{\mu_A(x)} = \overline{\mu_A(y)}\right)$.

Proof: Since in theorem 2, we prove that the measure $D_{O2}(x, y)$ is a valid measure so for first part for this lemma we have, $\left(\rho_{xy} + \sigma_{xy}\right) = 2$. Thus we have, $\sin\left[\frac{\pi}{4}\left(\rho_{xy} + \sigma_{xy}\right)\right] = \sin\frac{\pi}{2} = 1$. Hence $D_{O2sin}(x, y) = 1$ iff $(x = \langle 0, 0 \rangle)$ and $(y = \langle 1, 1 \rangle)$; or $(x = \langle 1, 1 \rangle)$ and $(y = \langle 0, 0 \rangle)$.

Now for second part of this lemma we have $\left(\rho_{xy} + \sigma_{xy}\right) = 0$. Thus we have $\sin\left[\frac{\pi}{4}\left(\rho_{xy} + \sigma_{xy}\right)\right] = 0$. Hence $D_{O2sin}(x, y) = 0$ iff $\left(\underline{\mu_A(x)} = \underline{\mu_A(y)}\right)$ and $\left(\overline{\mu_A(x)} = \overline{\mu_A(y)}\right)$.

Lemma 3: Let A be a fuzzy rough set in X, $x = \langle \underline{\mu_A(x)}, \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)}, \overline{\mu_A(y)} \rangle$ the fuzzy rough values in A and $D_{O2sin}(x, y)$ as above, then

a. $D_{O2sin}(x, y) = D_{O2sin}(y, x)$,
b. $D_{O2sin}(x, y) = D_{O2sin}(x^c, y^c)$,

Proof: Since in theorem 2, we prove that the measure $D_{O2}(x, y)$ is a valid measure so lemma is trivially proved.

Lemma 4: Let A be a fuzzy rough set in X, $x = \langle \underline{\mu_A(x)}, \overline{\mu_A(x)} \rangle, y = \langle \underline{\mu_A(y)}, \overline{\mu_A(y)} \rangle$ and $z = \langle \underline{\mu_A(z)}, \overline{\mu_A(z)} \rangle$ the fuzzy rough values in A and $D_{O1sin}(x, y)$ as above. If $\forall x \in \overline{A}, D_{O2sin}(x, y) = D_{O2sin}(x, z)$ then $D_{O2sin}(y, z) = 0$.

Proof: Since in theorem 2, we prove that the measure $D_{O2}(x,y)$ is a valid measure so we have $(\rho_{yz}+$ *σyz=0.* Thus we have sin*π4ρyz+σyz=0.* Hence *DO2siny, z=0, ∀x∈A, DO2sinx, y=DO2sinx,z.*

Hence proposed measure is valid measures

Similarly we prove the validity of part (b)

Theorem 5: Let A be a fuzzy rough set in X, $x=\langle \underline{\mu_A(x)}, \overline{\mu_A(x)}\rangle, y=\langle \underline{\mu_A(y)}, \overline{\mu_A(y)}\rangle$ the fuzzy rough values in A. Then the following measures of the degree of similarity between the fuzzy rough values x and y defined below are valid measures.

a. $S_{osin\,1}(x,y)=\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\ \rangle\right]$
b. $S_{ocos\,1}(x,y)=\cos\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\right]$
c. $S_{otan\,1}(x,y)=\tan\left[\frac{\pi}{4}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\ \rangle\right]$

Proof: First we prove part 'a' defined valid measures. To prove this measure is a valid similarity measure we must prove following lemmas:

Lemma 1: Let $A\in F^R(X)$ and $S_{osin\,1}(x,y)$ is defined as above. Since $x=\langle \underline{\mu_A(x)}, \overline{\mu_A(x)}\rangle, y=\langle \underline{\mu_A(y)}, \overline{\mu_A(y)}\rangle$ and $z=\langle \underline{\mu_A(z)}, \overline{\mu_A(z)}\rangle\in A$. If $x\le y\le z$, then $S_{osin\,1}(x,z)\le min\{S_{osin\,1}(x,y), S_{osin\,1}(y,z)\}$.

Proof: Since measure $S_Z(x,y)=1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)$ is a valid measure, So we have

$$1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(z)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(z)}\right|\right)\le min\left\{\begin{matrix}1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\\ 1-\frac{1}{2}\left(\left|\underline{\mu_A(y)}-\underline{\mu_A(z)}\right|+\left|\overline{\mu_A(y)}-\overline{\mu_A(z)}\right|\right)\end{matrix}\right\}$$

Since sine is an increasing function in interval $\left[0,\frac{\pi}{2}\right]$, so we have

$$\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(z)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(z)}\right|\right)\ \rangle\right]$$
$$\le min\left\{\begin{matrix}\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\ \rangle\right]\\ \sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A(y)}-\underline{\mu_A(z)}\right|+\left|\overline{\mu_A(y)}-\overline{\mu_A(z)}\right|\right)\ \rangle\right]\end{matrix}\right\}$$

Hence $S_{osin\,1}(x,z)\le min\{S_{osin\,1}(x,y), S_{osin\,1}(y,z)\}$, when $x\le y\le z$.

Lemma 2: Let $A\in F^R(X)$ and $S_{osin\,1}(x,y)$ as above. Then:

1. $S_{osin\,1}(x,y)=0\Leftrightarrow (x=\langle 0,0\rangle\ and\ y=\langle 1,1\rangle)\ or\ (x=\langle 1,1\rangle\ and\ y=\langle 0,0\rangle)$.
2. $S_{osin\,1}(x,y)=1\Leftrightarrow \underline{\mu_A(x)}=\underline{\mu_A(y)}\ and\ \overline{\mu_A(x)}=\overline{\mu_A(y)}$.

Proof: Since measure $S_Z(x,y)=1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)$ is a valid measure, so in first case we have $\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)=2$, then we have $\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\ \rangle\right]=0$. Thus we have $S_{osin\,1}(x,y)=0\Leftrightarrow (x=\langle 0,0\rangle\ and\ y=\langle 1,1\rangle)\ or\ (x=\langle 1,1\rangle\ and\ y=\langle 0,0\rangle)$. In second case we have $\left(\left|\underline{\mu_A(x)}-\underline{\mu_A(y)}\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)=0$, then

$\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A}(x)-\underline{\mu_A}(y)\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\rangle\right]=1.$ thus we have $S_{osin\,1}(x,y)=1 \Leftrightarrow \underline{\mu_A}(x)=\underline{\mu_A}(y)$ and $\overline{\mu_A(x)}=\overline{\mu_A(y)}$.

Lemma 3: Let $A \in F^R(X)$ and $S_{osin\,1}(x,y)$ as above. Then $S_{osin\,1}(x,y)=S_{osin\,1}(y,x)$ and $S_{osin\,1}(x,y)=S_{osin1}(x^c,y^c)$.

Proof: Since measure $S_Z(x,y)=1-\frac{1}{2}\left(\left|\underline{\mu_A}(x)-\underline{\mu_A}(y)\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)$ is a valid measure, so in first case we have $\left(\left|\underline{\mu_A}(x)-\underline{\mu_A}(y)\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)=\left(\left|\underline{\mu_A}(y)-\underline{\mu_A}(x)\right|+\left|\overline{\mu_A(y)}-\overline{\mu_A(x)}\right|\right)$, then we have $\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A}(x)-\underline{\mu_A}(y)\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)\rangle\right]=\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A}(y)-\underline{\mu_A}(x)\right|+\right.\right.$ *μA(y)−μA(x)* . Thus we have *Sosin1x, y=Sosin1y, x*. Similarly we prove *Sosin1x, y=Sosin1xc, yc.*

Lemma 4: Let $A \in F^R(X)$ and $S_{osin\,1}(x,y)$ as above. If $S_{osin\,1}(x,y)=S_{osin1}(x,z)\ \forall x \in X$, then $S_{osin\,1}(y,z)=1$.

Proof. Since measure $S_Z(x,y)=1-\frac{1}{2}\left(\left|\underline{\mu_A}(x)-\underline{\mu_A}(y)\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)$ is a valid measure, so we have $\left(\left|\underline{\mu_A}(y)-\underline{\mu_A}(z)\right|+\left|\overline{\mu_A(y)}-\overline{\mu_A(z)}\right|\right)=0$ if for all $x \in X$, $\left(\left|\underline{\mu_A}(x)-\underline{\mu_A}(y)\right|+\left|\overline{\mu_A(x)}-\overline{\mu_A(y)}\right|\right)=\left(\left|\underline{\mu_A}(y)-\underline{\mu_A}(z)\right|+\left|\overline{\mu_A(y)}-\overline{\mu_A(z)}\right|\right)$. Then we have $\sin\left[\frac{\pi}{2}\langle 1-\frac{1}{2}\left(\left|\underline{\mu_A}(y)-\underline{\mu_A}(z)\right|+\left|\overline{\mu_A(y)}-\right.\right.\right.$ *μA(z)* *=1,* if for all *x∈X,* sin*π21−12μA(x)−μA(y)+μA(x)−μA(y)* *=*sin*π21−12μA(x)−μA(z)+μA(x)−μA(z)* . Thus we have *Sosin1y, z=1,* when *Sosin1x, y=Sosin1x, z* $\forall x \in X$.

Since above all four lemmas satisfies the property of validity for a similarity measure of the fuzzy rough sets elements. Thus our proposed measure is a valid measure.

Similarly we prove part 'c' and since part 'b' is equivalent to part 'a' so it is also valid.

Theorem 5: Let A be a fuzzy rough set in X, $x=\langle\underline{\mu_A}(x),\ \overline{\mu_A(x)}\rangle$, $y=\langle\underline{\mu_A}(y),\ \overline{\mu_A(y)}\rangle$ the fuzzy rough values in A. Then the following measures of the degree of similarity between the fuzzy rough values x and y defined below are valid measures.

1. $S_{ocos\,2}(x,y)=\cos\left[\frac{\pi}{4}\left(\rho_{xy}+\sigma_{xy}\right)\right]$,
2. $S_{otan\,2}(x,y)=\tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}\left(\rho_{xy}+\sigma_{xy}\right)\right)\right]$.

Proof: Since measure defined in first part is equivalent to the measure defined in –(3.5) which is a valid measure hence our proposed measure is also a valid measure.

Now we prove second measure is a valid measure and we prove it by following lemmas

Lemma 1: Let $A \in F^R(X)$ and $S_{otan\,2}(x,y)$ is defined as above. Since $x=\langle\underline{\mu_A}(x),\ \overline{\mu_A(x)}\rangle$, $y=\langle\underline{\mu_A}(y),\ \overline{\mu_A(y)}\rangle$ and $z=\langle\underline{\mu_A}(z),\ \overline{\mu_A(z)}\rangle \in A$. If $x \le y \le z$, then $S_{otan\,2}(x,z) \le min\{S_{otan\,2}(x,y), S_{otan\,2}(y,z)\}$.

Proof: Since measure $S(x,y)=1-\frac{1}{2}\left(\rho_{xy}+\sigma_{xy}\right)$ is a valid measure, So we have

$$1-\frac{1}{2}(\rho_{xz}+\sigma_{xz}) \le min\begin{Bmatrix}1-\frac{1}{2}(\rho_{xy}+\sigma_{xy})\\ 1-\frac{1}{2}(\rho_{yz}+\sigma_{yz})\end{Bmatrix}$$

Since tan is an increasing function in interval $\left[0,\frac{\pi}{4}\right]$, so we have

$$\tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(\rho_{xz}+\sigma_{yz})\right)\right] \le min\begin{Bmatrix}\tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(\rho_{xy}+\sigma_{xy})\right)\right]\\ \tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(\rho_{yz}+\sigma_{yz})\right)\right]\end{Bmatrix}$$

Hence $S_{otan\,2}(x,z) \le min\{S_{otan\,2}(x,y), S_{otan\,2}(y,z)\}$, when $x \le y \le z$.

Lemma 2: Let $A \in F^R(X)$ and $S_{otan\,2}(x,y)$ as above. Then:

1. $S_{otan\,2}(x,y) = 0 \Leftrightarrow (x = \langle 0,0\rangle \; and \; y = \langle 1,1\rangle) \; or \; (x = \langle 1,1\rangle \; and \; y = \langle 0,0\rangle)$.
2. $S_{otan\,2}(x,y) = 1 \Leftrightarrow \underline{\mu_A(x)} = \underline{\mu_A(y)} \; and \; \overline{\mu_A(x)} = \overline{\mu_A(y)}$.

Proof: Since measure $S(x,y) = 1-\frac{1}{2}(\rho_{xy}+\sigma_{xy})$ is a valid measure, so in first case we have $(\rho_{xy} + \sigma xy=2,$ then we have tan$\pi 41-12\rho xy+\sigma xy=0$. Thus we have $Sotan2x,\; y=0\Leftrightarrow x=0,\; 0 \; and \; y=1,\; 1 \; or$ $(x = \langle 1,1\rangle \; and \; y = \langle 0,0\rangle)$. In second case we have $(\rho_{xy}+\sigma_{xy}) = 0$, then $\tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(\rho_{xy}+\sigma_{xy})\right)\right] = 1$. thus we have $S_{otan\,2}(x,y) = 1 \Leftrightarrow \underline{\mu_A(x)} = \underline{\mu_A(y)} \; and \; \overline{\mu_A(x)} = \overline{\mu_A(y)}$.

Lemma 3: Let $A \in F^R(X)$ and $S_{otan\,2}(x,y)$ as above. Then $S_{otan\,2}(x,y) = S_{otan\,2}(y,x)$ and $S_{otan\,2}(x,y) = S_{otan\,2}(x^c,y^c)$.

Proof: Since measure $S(x,y) = 1-\frac{1}{2}(\rho_{xy}+\sigma_{xy})$ is a valid measure, so in first case we have $(\rho_{xy} + \sigma xy=\rho yx+\sigma yx,$ then we have tan$\pi 41-12\rho xy+\sigma xy=tan\pi 41-12\rho yx+\sigma yx$. Thus we have $Sotan2x,\; y=Sotan2y,\; x$. Similarly we prove $Sotan2x,\; y=Sotan2xc,\; yc$.

Lemma 4: Let $A \in F^R(X)$ and $S_{otan\,2}(x,y)$ as above. If $S_{otan\,2}(x,y) = S_{otan\,2}(x,z) \; \forall x \in X$, then $S_{otan\,2}(y,z) = 1$.

Proof. Since measure $S(x,y) = 1-\frac{1}{2}(\rho_{xy}+\sigma_{xy})$ is a valid measure, so we have $(\rho_{yz}+\sigma_{yz}) = 0$ if for all $x \in X$, $(\rho_{xy}+\sigma_{xy}) = (\rho_{xz}+\sigma_{xz})$ Then we have $\tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(\rho_{yz}+\sigma_{yz})\right)\right] = 1$, if for all $x \in X$, $\tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(\rho_{xy}+\sigma_{xy})\right)\right] = \tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(\rho_{xz}+\sigma_{xz})\right)\right]$. Thus we have $S_{otan\,2}(y,z) = 1$, when $S_{otan\,2}(x,y) = S_{otan\,2}(x,z) \; \forall x \in X$.

Since above all four lemmas satisfies the property of validity for a similarity measure of the fuzzy rough sets elements. Thus our proposed measure is a valid measure.

5. Similarity & Distance Measures between Fuzzy Rough Sets:

Definition: Let $A, B \in F^R(X)$, $X = \{x_1, x_2, \dots \dots, x_m\}$. If $F_A^R(x) = \langle \underline{\mu_A(x)}, \; \overline{\mu_A(x)} \rangle$ is the fuzzy rough value of x in A and $F_B^R(x) = \langle \underline{\mu_B(x)}, \; \overline{\mu_B(x)} \rangle$ is the fuzzy rough value of x in B. Then the degree of similarity between the fuzzy rough sets A and B corresponding to the similarity measures defined in section 4 can be derived as

$$M_X(A,B) = \frac{1}{n}\sum_{j=1}^{n} S_X\left(F_A^R(x_j), F_B^R(x_j)\right) \qquad (5.1)$$

Zhang et al. [22] provide a similarity measure between two fuzzy rough sets as follows:

$$M_Z(A,B) = 1 - \frac{1}{2n}\sum_{j=1}^{n}\left(\left|\underline{\mu_A(x)} - \underline{\mu_A(y)}\right| + \left|\overline{\mu_A(x)} - \overline{\mu_A(y)}\right|\right)$$

Qi et al. [24] provide a similarity measure between two fuzzy rough sets as follows:

$$M_Q(A,B) = 1 - \frac{1}{2n}\sum_{j=1}^{n}\left(\left|\rho_A(x_j) - \rho_B(x_j)\right| + \left|\sigma_A(x_j) - \sigma_B(x_j)\right|\right)$$

Sharma and Gupta [25] provide sine trigonometric similarity measure between two fuzzy rough set as follows:

$$M_{SG}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\sin\left[\frac{\pi}{2}\left(1 - \frac{\left(\rho_A(x_j) - \rho_B(x_j)\right)}{2} - \frac{\left(\sigma_A(x_j) - \sigma_B(x_j)\right)}{2}\right)\right]$$

Now the similarity measures between two set corresponding to measure defined in (4.2(a)) is

$$M_{osin\,1}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\sin\left[\frac{\pi}{2}\langle 1 - \frac{1}{2}\left(\left|\underline{\mu_A(x_j)} - \underline{\mu_B(x_j)}\right| + \left|\overline{\mu_A(x_j)} - \overline{\mu_B(x_j)}\right|\right)\rangle\right] \qquad (5.2)$$

Now the similarity measures between two set corresponding to measure defined in (4.2(b)) is

$$M_{ocos\,1}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x_j)} - \underline{\mu_B(x_j)}\right| + \left|\overline{\mu_A(x_j)} - \overline{\mu_B(x_j)}\right|\right)\right] \qquad (5.3)$$

Now the similarity measures between two set corresponding to measure defined in (4.3) is

$$M_{otan\,1}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\tan\left[\frac{\pi}{4}\langle 1 - \frac{1}{2}\left(\left|\underline{\mu_A(x_j)} - \underline{\mu_B(x_j)}\right| + \left|\overline{\mu_A(x_j)} - \overline{\mu_B(x_j)}\right|\right)\rangle\right] \qquad (5.3)$$

Now the similarity measures between two set corresponding to measure defined in (4.7) is

$$M_{otan\,2}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\tan\frac{\pi}{4}\left(1 - \left|\frac{\left(\rho_A(x_j) - \rho_B(x_j)\right)}{2}\right| - \left|\frac{\left(\sigma_A(x_j) - \sigma_B(x_j)\right)}{2}\right|\right) \qquad (5.4)$$

Now the similarity measures between two set corresponding to measure defined in (4.8) is

$$M_{ocos\,2}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\frac{\pi}{4}\left(\left|\left(\rho_A(x_j) - \rho_B(x_j)\right)\right| + \left|\left(\sigma_A(x_j) - \sigma_B(x_j)\right)\right|\right) \qquad (5.5)$$

where $\rho_A(x_j) = \underline{\mu_A(x_j)} + \left(\overline{\mu_A(x_j)} - \underline{\mu_A(x_j)}\right)\underline{\mu_A(x_j)}$ and $\sigma_A(x_j) = 1 - \overline{\mu_A(x_j)} + \left(\overline{\mu_A(x_j)} -\right.$ *μA(xj)1− μAxj.*

It is clear that $M_X(A,B) \in [0,1]$, and A and B are more similar for superior value of $M_X(A,B)$.

The following inferences are evident.

Proposition 1. $M_X(A,B) = M_X(B,A),\;\; M_X(A,B) = M_X(A^c,B^c)$.

Proposition2. $M_X(A,B) = 0 \Leftrightarrow \left(A = \sum_{j=1}^{m}\langle 0,0\rangle/x_j \text{ and } B = \sum_{j=1}^{m}\langle 1,1\rangle/x_j\right) \text{ or } \left(A = \sum_{j=1}^{m}\langle 1,1\rangle/\right.$ *xj and B =j=1m0, 0/xj.*

Proposition 3. $M_X(A,B) = 1 \Leftrightarrow \overline{\mu_A(x_j)} = \overline{\mu_B(x_j)} \text{ and } \underline{\mu_A(x_j)} = \underline{\mu_B(x_j)}, \forall x_j \in X$.

We may define the order relation between the fuzzy rough sets:

$$A \subseteq B \Leftrightarrow \underline{\mu_A(x)} \le \underline{\mu_B(x)} \text{ and } \overline{\mu_A(x)} \le \overline{\mu_B(x)}, \forall x_j \in X$$

Proposition 4: $\forall A,B,C \in F^R(X),\;\; A \subseteq B \subseteq C \Longrightarrow M_X(A,C) \le min\{M_X(A,B), M_X(B,C)\}$

Since above measures satisfy all the condition of similarity measure so these measures are valid measures.

Definition: Let $A,B \in F^R(X)$, $X = \{x_1, x_2, \dots \dots, x_m\}$. If $F_A^R(x) = \langle \underline{\mu_A(x)},\; \overline{\mu_A(x)}\rangle$ is the fuzzy rough value of x in A and $F_B^R(x) = \langle \underline{\mu_B(x)},\; \overline{\mu_B(x)}\rangle$ is the fuzzy rough value of x in B. Then the degree of dissimilarity between the fuzzy rough sets A and B corresponding to the distance measures defined in section 4 can be derived as

$$Z_X(A,B) = \frac{1}{n}\sum_{j=1}^{n} D_X\left(F_A^R(x_j), F_B^R(x_j)\right) \qquad (5.6)$$

Now the distance measures between two set corresponding to measure defined in (4.1) is

$$Z_{o1}(A,B) = \frac{1}{2n}\sum_{j=1}^{n}\left(\left|\underline{\mu_A(x_j)} - \underline{\mu_B(x_j)}\right| + \left|\overline{\mu_A(x_j)} - \overline{\mu_B(x_j)}\right|\right) \qquad (5.7)$$

Now the distance measures between two set corresponding to measure defined in (4.4) is

$$Z_{o1sin}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\sin\left[\frac{\pi}{4}\left(\left|\underline{\mu_A(x_j)} - \underline{\mu_B(x_j)}\right| + \left|\overline{\mu_A(x_j)} - \overline{\mu_B(x_j)}\right|\right)\right] \qquad (5.8)$$

Now the distance measures between two set corresponding to measure defined in (4.5) is

$$Z_{o1tan}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\tan\left[\frac{\pi}{8}\left(\left|\underline{\mu_A(x_j)} - \underline{\mu_B(x_j)}\right| + \left|\overline{\mu_A(x_j)} - \overline{\mu_B(x_j)}\right|\right)\right] \qquad (5.9)$$

Now the distance measures between two set corresponding to measure defined in (4.6) is

$$Z_{o2}(A,B) = \frac{1}{2n}\sum_{j=1}^{n}\left(\left|\rho_A(x_j) - \rho_B(x_j)\right| + \left|\sigma_A(x_j) - \sigma_B(x_j)\right|\right) \qquad (5.10)$$

Now the distance measures between two set corresponding to measure defined in (4.9) is

$$Z_{osin\,2}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\sin\left[\frac{\pi}{4}\left(\left|\rho_A(x_j) - \rho_B(x_j)\right| + \left|\sigma_A(x_j) - \sigma_B(x_j)\right|\right)\right] \qquad (5.11)$$

Now the distance measures between two set corresponding to measure defined in (4.10) is

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$$Z_{otan\,2}(A,B)=\frac{1}{n}\sum_{j=1}^{n}\tan\left[\frac{\pi}{8}\left(\left|\rho_A(x_j)-\rho_B(x_j)\right|+\left|\sigma_A(x_j)-\sigma_B(x_j)\right|\right)\right] \quad (5.11)$$

where $\rho_A(x_j)=\underline{\mu_A(x_j)}+\left(\overline{\mu_A(x_j)}-\underline{\mu_A(x_j)}\right)\underline{\mu_A(x_j)}$ and $\sigma_A(x_j)=1-\overline{\mu_A(x_j)}+\left(\overline{\mu_A(x_j)}-\right.$ *μA(xj)1−μAxj.*

It is clear that $Z_X(A,B)\in[0,1]$, and A and B are more dissimilar for superior value of $Z_X(A,B)$.

The following inferences are evident.

Proposition 1. $Z_X(A,B)=Z_X(B,A),\ \ Z_X(A,B)=Z_X(A^c,B^c)$.

Proposition2. $Z_X(A,B)=1 \Leftrightarrow \left(A=\sum_{j=1}^{m}\langle 0,0\rangle/x_j \text{ and } B=\sum_{j=1}^{m}\langle 1,1\rangle/x_j\right)$ or $\left(A=\sum_{j=1}^{m}\langle 1,1\rangle/\right.$ *xj and B=j=1m0, 0/xj.*

Proposition 3. $Z_X(A,B)=0 \Leftrightarrow \overline{\mu_A(x_j)}=\overline{\mu_B(x_j)}$ and $\underline{\mu_A(x_j)}=\underline{\mu_B(x_j)},\forall x_j\in X$.

We may define the order relation between the fuzzy rough sets:

$$A\subseteq B \Leftrightarrow \underline{\mu_A(x)}\le\underline{\mu_B(x)} \text{ and } \overline{\mu_A(x)}\le\overline{\mu_B(x)},\forall x_j\in X$$

Proposition 4: $\forall A,B,C\in F^R(X),\ \ A\subseteq B\subseteq C \Longrightarrow Z_X(A,C)\ge max\{Z_X(A,B),Z_X(B,C)\}$

Since above measures satisfy all the condition of distance measure so these measures are valid measures.

6. Application and Comparison of Proposed and Existing Measures:

Here we expound the application of suggested measures by simulated data onto a hiring decision problem. The dilemma is involved in 4 applicants (A_1,A_2,A_3,A_4) for a position; everyone is appraised over five characteristics, which are experience in the specific job function (a), educational background (b), adaptability (c), aptitude for team work (d), personality (e). For every applicant we establish FRSs function by fuzzy method or probability method and obtain their attribute values represented in table 1.

	a	b	c	d	e
A_1	[0.4, 0.6]	[0.3, 0.7]	[0.5, 0.9]	[0.5, 0.8]	[0.6, 0.8]
A_2	[0.2, 0.4]	[0.3, 0.5]	[0.2, 0.3]	[0.7, 0.9]	[0.8, 1]
A_3	[0.1, 0.1]	[0, 0]	[0.2, 0.3]	[0.1, 0.2]	[0.6, 0.6]
A_4	[0.8, 0.8]	[0.9, 1]	[1, 1]	[0.7, 0.8]	[0.6, 0.6]

Table 1: Attribute sets of the applicants

The optimal attributes values of applicants which are used as a referenced to compare the existing one is represent by A, as $A=\{[0.3,0.5],[0.4,0.6],[0.6,0.8],[0.5,0.9],[0.9,1]\}$.

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	a	b	c	d	e
$\underline{\mu_{A_1}}$	0.40	0.30	0.50	0.50	0.60
$\overline{\mu_{A_1}}$	0.60	0.70	0.90	0.80	0.80
τ_{A_1}	0.20	0.40	0.40	0.30	0.20
ρ_{A_1}	0.48	0.42	0.70	0.65	0.72
σ_{A_1}	0.48	0.42	0.14	0.26	0.24

	a	b	c	d	e
$\underline{\mu_{A_2}}$	0.20	0.30	0.20	0.70	0.80
$\overline{\mu_{A_2}}$	0.40	0.50	0.70	0.90	1.00
τ_{A_2}	0.20	0.20	0.50	0.20	0.20
ρ_{A_2}	0.24	0.36	0.30	0.84	0.96
σ_{A_2}	0.72	0.60	0.45	0.12	0.00

	a	b	c	d	e
$\underline{\mu_{A_3}}$	0.10	0.00	0.20	0.10	0.20
$\overline{\mu_{A_3}}$	0.10	0.00	0.30	0.20	0.20
τ_{A_3}	0.00	0.00	0.10	0.10	0.00
ρ_{A_3}	0.10	0.00	0.22	0.11	0.20
σ_{A_3}	0.30	1.00	0.77	0.88	0.80

	a	b	c	d	e
$\underline{\mu_{A_4}}$	0.80	0.90	1.00	0.70	0.60
$\overline{\mu_{A_4}}$	0.80	1.00	1.00	0.80	0.60
τ_{A_4}	0.00	0.10	0.00	0.10	0.00
ρ_{A_4}	0.80	0.99	1.00	0.77	0.60
σ_{A_4}	0.20	0.00	0.00	0.22	0.40

	a	b	c	d	e
$\underline{\mu_A}$	0.30	0.40	0.60	0.50	0.90
$\overline{\mu_A}$	0.50	0.60	0.80	0.90	1.00
τ_A	0.20	0.20	0.20	0.40	0.10
ρ_A	0.36	0.48	0.72	0.70	0.99
σ_A	0.60	0.48	0.24	0.14	0.00

Using the proposed and existing similarity measures we find the suitable applicant for the job in given applicants. For this we find the similarity between standard set A and given four applicant sets. For comparison, we show the final results of the above similarity measures as in table 2.

	(A, A_1)	(A, A_2)	(A, A_3)	(A, A_4)	Order
$M_Z(A, A_i)$	0.88	0.88	0.49	0.67	$A_1 = A_2 > A_4 > A_3$
$M_{osin\,1}(A, A_i)$	0.976	0.976	0.678	0.857	$A_1 = A_2 > A_4 > A_3$
$M_{ocos\,1}(A, A_i)$	0.976	0.976	0.678	0.857	$A_1 = A_2 > A_4 > A_3$
$M_{otan\,1}(A, A_i)$	0.83	0.83	0.411	0.586	$A_1 = A_2 > A_4 > A_3$
$M_Q(A, A_i)$	0.884	0.87	0.449	0.671	$A_1 > A_2 > A_4 > A_3$
$M_{SG}(A, A_i)$	0.977	0.967	0.624	0.845	$A_1 > A_2 > A_4 > A_3$
$M_{ocos\,2}(A, A_i)$	0.977	0.962	0.723	0.845	$A_1 > A_2 > A_4 > A_3$
$M_{otan\,2}(A, A_i)$	0.836	0.788	0.446	0.593	$A_1 > A_2 > A_4 > A_3$

Table 2. Similarity degree of different similarity measures between applicants

Using the proposed distance measures we find the suitable applicant for the job in given applicants. For this we find the dissimilarity between standard set A and given four applicant sets. For comparison, we show the final results of the above distance measures as in table 3.

	(A, A_1)	(A, A_2)	(A, A_3)	(A, A_4)	Order
$Z_{o1}(A, A_i)$	0.12	0.12	0.51	0.33	$A_1 = A_2 < A_4 < A_3$
$Z_{osin\,1}(A, A_i)$	0.186	0.186	0.698	0.489	$A_1 = A_2 < A_4 < A_3$
$Z_{otan\,1}(A, A_i)$	0.094	0.094	0.43	0.266	$A_1 = A_2 < A_4 < A_3$
$Z_{o2}(A, A_i)$	0.116	0.154	0.471	0.329	$A_1 < A_2 < A_4 < A_3$
$Z_{osin\,2}(A, A_i)$	0.179	0.237	0.66	0.482	$A_1 < A_2 < A_4 < A_3$
$Z_{otan\,2}(A, A_i)$	0.091	0.122	0.392	0.267	$A_1 < A_2 < A_4 < A_3$

Table 3. Dissimilarity degree of different distance measures between applicants

From table 2 we see that for similarity measures $M_Z(A, A_i)$, $M_{osin\,1}(A, A_i)$, $M_{ocos\,1}(A, A_i)$ and $M_{otan\,1}(A, A_i)$, we have applicants order as $A_1 = A_2 > A_4 > A_3$, thus here in taking decision difficulty arises because applicants A_1& A_2 has same standard. It shows that the similarity measures corresponding to Zhang et al. [22] proposed measures are not suitable for making decision. While for similarity measures $M_Q(A, A_i)$, $M_{SG}(A, A_i)$, $M_{ocos\,2}(A, A_i)$ and $M_{otan\,2}(A, A_i)$, we have applicants order as $A_1 > A_2 > A_4 > A_3$, here we prefer applicant A_1, because he has high similarity value with respect to standard set value of applicants.

Similarly, from table 3 we see that for distance measures $Z_{o1}(A, A_i)$, $Z_{osin\,1}(A, A_i)$ and $Z_{otan\,1}(A, A_i)$, we have applicants order as $A_1 = A_2 < A_4 < A_3$, thus here in taking decision difficulty arises because applicants A_1& A_2 have same standard. It shows that the distance measures corresponding to Zhang et al. [22] proposed measures are not suitable for making decision. While for distance measures $Z_{o2}(A, A_i)$, $Z_{0sin\,2}(A, A_i)$ and $Z_{otan\,2}(A, A_i)$, we have applicants order as $A_1 < A_2 < A_4 < A_3$, here we prefer applicant A_1, because he has low dissimilarity values with respect to standard set value of applicants

The comparison of similarity measures are shown in fig. 1. Here series 1 to series 4 shows the similarity value of sets (A, A_1) to (A, A_4), and number 1 to 8 shows the different similarity measures as order shown in table 2 respectively.

The comparison of distance measures are shown in fig. 2. Here series 1 to series 4 shows the dissimilarity value of sets (A, A_1) to (A, A_4), and number 1 to 6 shows the different distance measures as order shown in table 3 respectively.

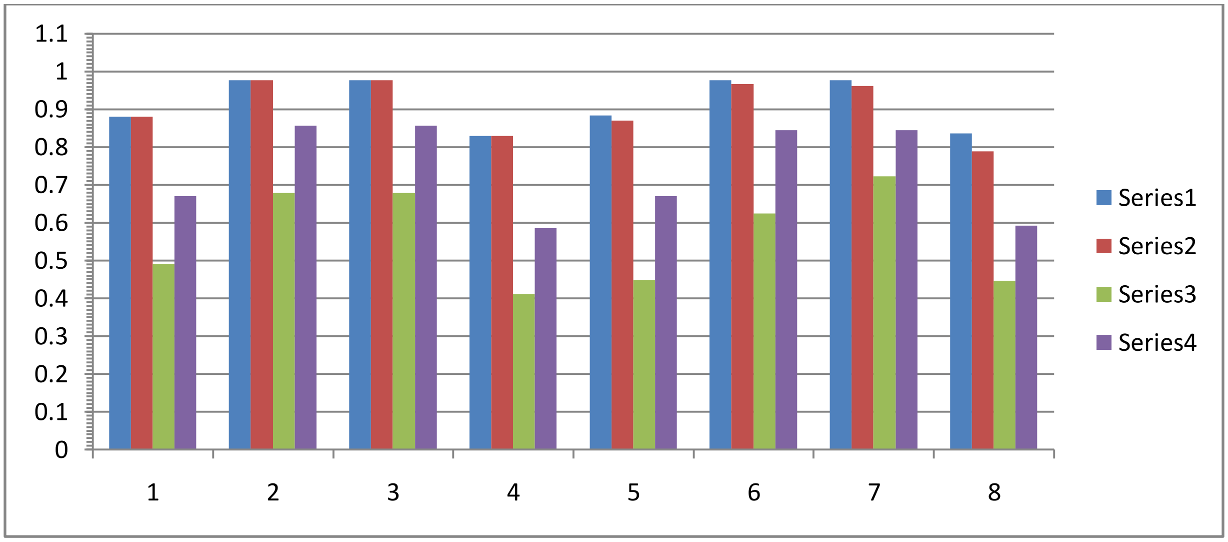

Fig. 1: The comparison chart of different similarity measures discussed above

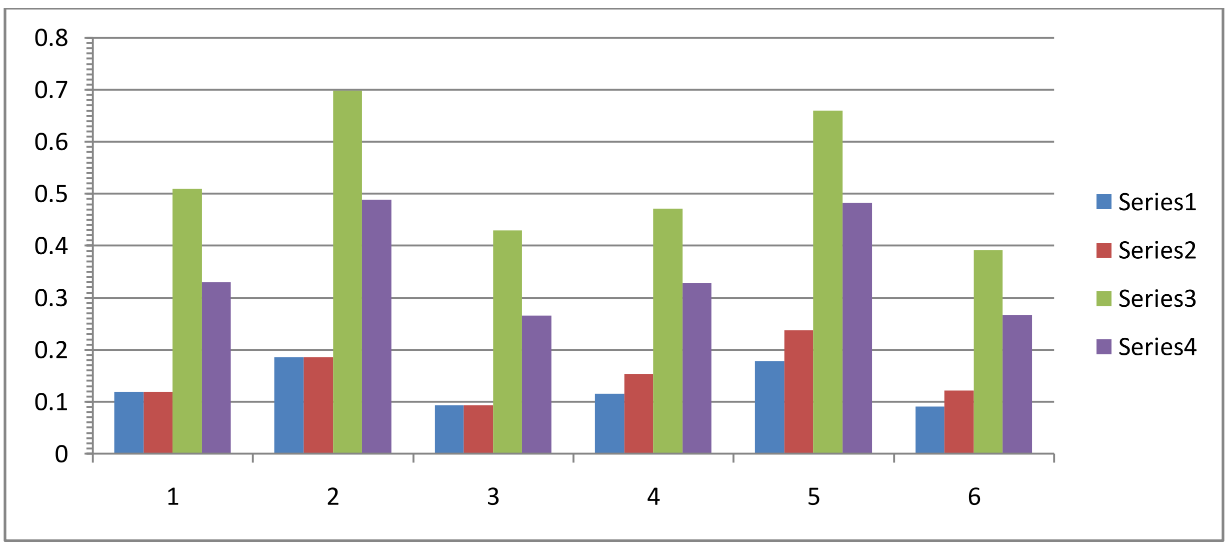

Fig. 2: The comparison chart of different distance measures discussed above

7. Conclusion:

Similarity and distance measures are good tools in the concept of vagueness and inexactness. Both are complementary to each other. We can derive one measure with the use of other measure. Using this concept here we propose some distance measures between fuzzy rough sets and their element by using existing similarity measures. Corresponding to these proposed measures we also propose some trigonometric distance measures. Here we also proposed some trigonometric similarity measures between fuzzy rough sets and their elements. Also show their validity. At last we discussed the application of proposed measures in decision making problems and compare them. Here we also conclude that the similarity and distance measures which are proposed by using Qi et al [24] is more suitable than Zhang et al [22].

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